EQUIVALENCE OF FOURIER SPECTROSCOPY AND SLOW PASSAGE IN NUCLEAR MAGNETIC RESONANÇE

R. R. ERNST, W. P. AUE, E. BARTHOLDI, A. HÖHENER and S. SCHÄUBLIN

Laboratorium für Physikalische Chemie, Eidg. Technische Hochschule, 8006 Zürich, Switzerland

ABSTRACT

The conditions for the equivalence of Fourier spectroscopy and of slow passage experiments in nuclear magnetic resonance are formulated. The application to several critical cases is discussed, including chemically induced dynamic nuclear polarization, Overhauser experiments, double resonance and systems with chemical exchange.

1. INTRODUCTION

It is well known that n.m.r.–Fourier spectroscopy offers considerable advantages over the conventional slow passage techniques[1-5]. First of all, sensitivity may be considerably higher. Additionally, its versatility is greater as all spectral parameters, including relaxation times, can be measured with the same general technique[3-8].

In most cases, it is tacitly assumed that the free induction decay is the Fourier transform of the low-power slow passage spectrum and that the information contained in both is the same, except for the different sensitivity. On the other hand, it is well known that the theoretical equivalence may be impeded in practical situations for the following possible reasons:

(i) Any sweep experiment is only an approximation to a true slow passage experiment as a finite time is available only for scanning the complete spectrum. This causes asymmetric signal distortions (in the extreme case wiggles) which are absent in Fourier spectroscopy[2].

(ii) Any sweep experiment requires a finite signal amplitude to detect the resonances in the background random noise. Deviations from a true linear response of the inherently non-linear spin system may occur (saturation effects).

(iii) Fourier spectroscopy involves digital data handling processes based on a sampled and digitized representation of the free induction decay. This representation may not be faithful in the sense that a complete reconstruction

47

of the original signal is impossible[9, 10]. Particularly, it is always necessary to limit the number of samples.

(*iv*) To obtain good sensitivity, Fourier spectroscopy requires a repetitive application of radio frequency pulses to excite a sequence of free induction decays. Interference effects between different decays cannot be completely avoided. This may cause inaccurate signal intensities and distorted line shapes[11, 12].

(*v*) The radio frequency pulse is necessarily of finite length and does not affect the entire spectrum equally. This again may lead to amplitude and phase anomalies[13].

All these effects cannot be interpreted as contradictions to the general equivalence of Fourier spectroscopy and slow passage. They are merely experimental artifacts and will not be discussed further in this paper.

There can be more fundamental objections to the basic equivalence which depend on the particular structure of the system under investigation. Equivalence can be proved for some restrictive conditions only. The formulation of these conditions and the discussion of their implications on practical measurements are the subjects of the following sections.

Two early proofs of the Fourier transform relationship between free induction decay and slow passage spectrum are given by Lowe and Norberg[14] and by Abragam[15]. Both of them are limited to some special cases.

2. CONDITIONS FOR EQUIVALENCE

The following meaning is given to the term 'equivalence': Fourier spectroscopy applied to a particular system is equivalent to the corresponding low-power slow passage experiment when the Fourier transform of the free induction decay signal for an arbitrary flip angle α is proportional to the slow passage spectrum.

The conditions to be fulfilled for equivalence require a particular form of the density operator equation for the investigated system. In many practical applications it is possible to represent this equation in the form:

$$\dot{\sigma} = -i[\mathcal{H}, \sigma] - \Gamma\{\sigma - \sigma_0\} \tag{1}$$

Here, $\mathcal{H}$ is the Hamiltonian (which may be time dependent), Γ is the relaxation superoperator and σ_0 is the equilibrium density operator. A more general form of the density operator equation which comprises all conceivable cases of practical interest is:

$$\dot{\sigma} = \Lambda\sigma - Q \tag{2}$$

where Λ is an arbitrary, possibly non-linear and time-dependent superoperator which describes the magnetic and electric interactions as well as relaxation. Q is an operator which is related to the steady-state conditions.

To prove equivalence, the following conditions are sufficient and, except for some special cases, also necessary:

(*i*) *The density operator equation must be time independent.* The superoperator Λ and the operator Q are not allowed to be functions of time.

(ii) *The density operator equation must be linear in σ,* i.e.

$$\Lambda a\sigma = a\Lambda\sigma, \text{ for an arbitrary constant } a \tag{3}$$

(iii) *The relevant part $\sigma_r(+0)$ of the density operator immediately after the rf pulse, $\sigma(+0)$, must be independent of the flip angle α,* except for a possible scaling factor:

$$\sigma(+0) = \sigma_r(+0) + \sigma_i(+0) \tag{4}$$

A formal proof of these statements will be given on another occasion. Some plausibility arguments are indicated here only.

(*i*) For a time-dependent density operator equation, the response to the applied rf pulse will depend on the particular instant at which the pulse was applied. A strict equivalence cannot hold in this case.

For any time-independent system, the free induction decay signal for a sufficiently small flip angle α is the Fourier transform of the slow passage signal, as the system may be approximated by a linearized density operator equation where the equivalence clearly holds. This is well known from the theory of linear systems[16, 17]. For the general equivalence it is thus sufficient to prove that the free induction decay signal is independent of the flip angle except for an arbitrary scaling factor.

(*ii*) It is clear that for a density operator equation, non-linear in the density operator, the free induction decay signal will change its shape as a function of the flip angle which determines its amplitude. A general equivalence cannot exist for a non-linear density operator equation.

It should be observed that the claim for linearity concerns exclusively the density operator. It is well known that with regard to input–output relations any spin system is non-linear as evidenced by possible saturation effects. This inherent non-linearity is not important in the present respect since during the free induction decay the Hamiltonian remains unperturbed.

(*iii*) The relevant part $\sigma_r(+0)$ is that part of $\sigma(+0)$ which may contribute during the free induction decay to the expectation value of the particular observable X, which is usually a component of the transverse magnetization, F_x or F_y. The relevant part may also be defined by means of the eigenoperators R_k of Λ:

$$\Lambda R_k = \lambda_k R_k \tag{5}$$

$\sigma_r(+0)$ is a linear combination of those eigenoperators R_k^r which are not orthogonal to the observable operator X:

$$\text{Tr}\{R_k^r X\} \neq 0 \tag{6}$$

The observed expectation value $\langle X \rangle$ is given by

$$\langle X \rangle(t) = \text{Tr}\{\sigma_r(t) X\} \tag{7}$$

where $\sigma_r(t)$ is determined by $\sigma_r(+0)$ exclusively. It follows that eigenoperators are constants of motion during the free induction decay. Therefore R_k^r are the only operators which may contribute to $\langle X \rangle(t)$. For an expectation value independent of the flip angle, the initial relevant part of the density operator must be independent of the flip angle as well. It is particularly

49

noted that the initial density operator $\sigma_r(+0)$ determines the integrated intensities of the resonance lines.

The somewhat abstract condition (*iii*) may be replaced by the following four more restrictive conditions which imply the condition (*iii*) but which are not necessary conditions for equivalence:

(*iiia*) *The system is in internal thermodynamic equilibrium* immediately before the applied rf pulse, i.e. it can be described by a spin temperature $\beta_s = 1/kT_s$ and

$$\sigma(-0) = \exp\{-\beta_s \mathscr{H}\}/\mathrm{Tr}[\exp\{-\beta_s \mathscr{H}\}] \tag{8}$$

T_s may differ from the environment temperature T.

(*iiib*) *The high-temperature approximation is fulfilled.* It implies that the density operator never deviates appreciably from unity and that it can be expanded in terms of the inverse temperature $\beta = 1/kT$:

$$\sigma(t) \approx 1/\mathrm{Tr}\{1\} + \beta\sigma_1(t) + O(\beta^2) \tag{9}$$

(*iiic*) *The high-field approximation is fulfilled.* This condition states that the dominant interaction in the system is the Zeeman interaction. Thus it is the only interaction which must be considered for the computation of the equilibrium density operator σ_0. If additionally condition (*iiib*) is assumed, one obtains:

$$\sigma_0 \approx \{1 + \beta \sum_k \gamma_k (1 - \delta_k) I_{zk} H_0\}/\mathrm{Tr}\{1\} \tag{10}$$

(*iiid*) *A homonuclear part of the complete system is excited exclusively* by the rf pulse. Chemical shift differences of the involved nuclei must be sufficiently small. The flip angle α_k experienced by a particular nuclear spin k is given by

$$\alpha_k = -\gamma_k (1 - \delta_k) H_1 \tau \tag{11}$$

where γ_k is the corresponding gyromagnetic ratio and δ_k the chemical shielding constant. H_1 is the strength of the rotating rf field. The flip angle will vary from nucleus to nucleus unless a homonuclear sub-system with similar chemical shifts is examined.

Condition (*iiib*) also implies condition (*ii*) as it is possible to expand the density operator and to neglect quadratic and higher terms in β. This linearizes the density operator equation.

Taking into account conditions (*iiia–d*), it is possible to write the density operator $\sigma(-0)$ before the rf pulse in the simple form:

$$\sigma(-0) \approx \{1 - \beta_s \hbar \Omega F_z\}/\mathrm{Tr}\{1\} \tag{12}$$

where Ω is an 'average resonance frequency' and $F_z = \sum_k I_{kz}$. Then one obtains for $\sigma(+0)$ assuming a rotation by the angle α about the x axis:

$$\sigma(+0) = e^{-iF_x\alpha} \sigma(-0) e^{+iF_x\alpha}$$

$$= \{1 - \beta_s \hbar \Omega (F_z \cos\alpha - F_y \sin\alpha)\} \frac{1}{\mathrm{Tr}\{1\}} \tag{13}$$

of which the relevant part for the observable F_y is:

$$\sigma_r(+0) = +\beta_s \hbar\Omega F_y \sin\alpha/\mathrm{Tr}\{1\} \tag{14}$$

This expression is, except for the factor $\sin\alpha$, independent of the flip angle. For the derivation of this equation the fact has been utilized that it is possible to neglect in the Hamiltonian and the relaxation superoperator all non-secular terms such that F_z can never contribute at a later instant to the transverse magnetization. This proves the general equivalence for this special case.

There are systems which violate conditions (*iiia*), (*iiib*) or (*iiic*) and still obey the general equivalence of Fourier spectroscopy and slow passage. The most obvious example is the one-spin system with spin quantum number 1/2. Thus in natural abundance ^{13}C and ^{15}N resonance with simultaneous proton decoupling the equivalence will always be fulfilled.

The conditions which are most likely violated in practical applications are conditions (*i*) for systems with time dependence, e.g. double resonance and chemical reactions, and condition (*iii*) for systems which are not in thermodynamic equilibrium, e.g. CIDNP and Overhauser polarization experiments. There has also been some discussion about possible non-linearities violating condition (*ii*). The following sections describe some of these cases in more detail.

3. FOURIER SPECTROSCOPY IN CIDNP EXPERIMENTS

The application of Fourier techniques to chemically induced nuclear polarization experiments[18,19] seems to be obvious as it is required to take snapshots of a non-equilibrium population of the energy levels caused by spin-selective chemical reactions.

In low-power slow passage experiments the signal intensities are just given by the population difference $P_k - P_j$ of the connected energy levels:

$$L_{jk}^{(SP)} \propto (P_k - P_j) \times |F_{y\cdot jk}|^2 \tag{15}$$

On the other hand, for Fourier experiments the integrated intensity of a non-degenerate transition is given by:

$$L_{jk}^{(FT)} = 2\mathrm{Re}\{\sigma(+0)_{jk} F_{ykj}\} \tag{16}$$

where the matrix element $\sigma(+0)_{jk}$ is given by:

$$\sigma(+0)_{jk} = \sum_r (e^{-iF_x\alpha})_{jr} (e^{+iF_x\alpha})_{rk} P_r \tag{17}$$

All matrix elements are taken in the eigenbase of the Hamiltonian. Here it has been assumed that the density operator before the rf pulse is diagonal in the eigenbase. There may be situations where this is not the case, as will be discussed on another occasion.

It can be derived from equation 16 that in general any line intensity in a Fourier CIDNP experiment will be affected by all populations P_r, except for a sufficiently small flip angle where the relative intensities are identical to those of a slow passage experiment and are given by equation 15. For
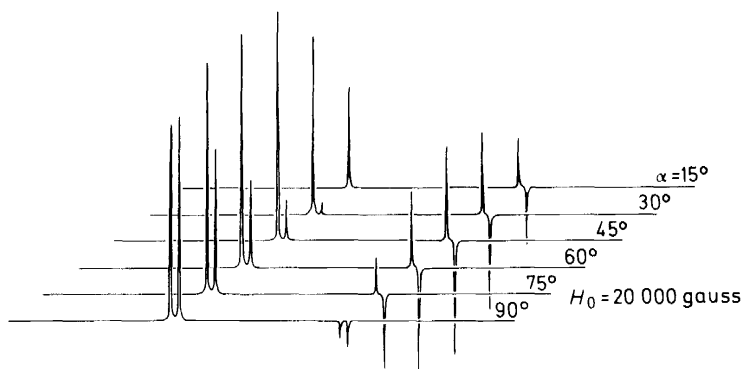
Figure 1. Computer simulation of Fourier CIDNP spectra for an AB system for different flip angles α. The following parameter values were used: magnetic field strength $H_0 = 20\,000$ gauss, difference of the g factors of the two radicals $\Delta g = 10^{-3}$; hyperfine coupling constants $A_1 = -40$ gauss, $A_2 = -60$ gauss; chemical shift difference $\Delta f = 40$ Hz; and spin-spin coupling constant $J = 2$ Hz. The computation was based on Ref. 20

large flip angles the intensities of a Fourier CIDNP experiment may significantly deviate from those of the corresponding slow passage experiment.

The most unfavourable case is $\alpha = 90°$. Here the mixing of the signal intensities is at a maximum. It is, for example, possible to show that for a weakly coupled spin system without equivalent nuclei, a $90°$ pulse causes all lines within a multiplet to have the same average intensity. It is well known that in the case of a pure multiplet effect this average intensity is zero. In this case, all resonance lines would disappear for a $90°$ pulse. On the other hand, a possible net effect would remain unaffected. The maximum signal intensity is often obtained for α near $45°$.

Computed examples of a chemically polarized AB system are given in *Figures 1* and *2*. The two sets of spectra differ by the utilized magnetic field strength. *Figure 1* shows a typical weakly coupled spin system, obtained at high fields, whereas the example in *Figure 2* is a strongly coupled system
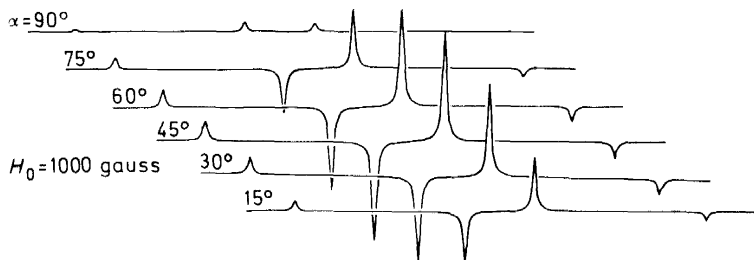

Figure 2. Computer simulation of Fourier CIDNP spectra for an AB system for different flip angles α. The same parameter values were used as in *Figure 1* except for the lower magnetic field strength $H_0 = 1000$ gauss and the corresponding chemical shift difference $\Delta f = 2$ Hz

52

which corresponds to a low magnetic field. The chemically induced nuclear polarization was computed using Adrian's diffusion model[20]. No relaxation effects were considered. The spectra for $\alpha = 15°$ nearly correspond to the spectra obtained in a slow passage experiment. The high-field example of *Figure 1* shows that one of the A lines has zero intensity. This can be explained by a compensation of a multiplet and net effect. On the other hand, the B multiplet is characterized by a strong multiplet effect. For a flip angle of 90°, the intensities within each multiplet become essentially equal and are determined exclusively by the net effect.

The example of *Figure 2* at a low magnetic field exhibits a dominant multiplet effect. The spectrum is nearly antisymmetric at small flip angle. For a 90° flip angle the intensities are all of the same sign and are very small. It is apparent that the most dramatic changes occur between $\alpha = 75°$ and $\alpha = 90°$. This is a feature of this particular example and should not be generalized.

These examples demonstrate clearly that Fourier CIDNP experiments must employ unusually small flip angles to obtain the same intensities as in slow passage. Already flip angles of 20–30° can give misleading perturbations of the intensities. On the other hand, when net effects only are desired, it is possible to eliminate multiplet effects by means of a 90° pulse. A detailed study of Fourier CIDNP experiments will be published shortly[21].

It is obvious that the mentioned difficulties can only occur in coupled spin systems. Effects of this kind will not appear in proton-decoupled ^{13}C and ^{15}N spectra where the equivalence will always be fulfilled.

4. FOURIER MEASUREMENTS OF OVERHAUSER EFFECTS

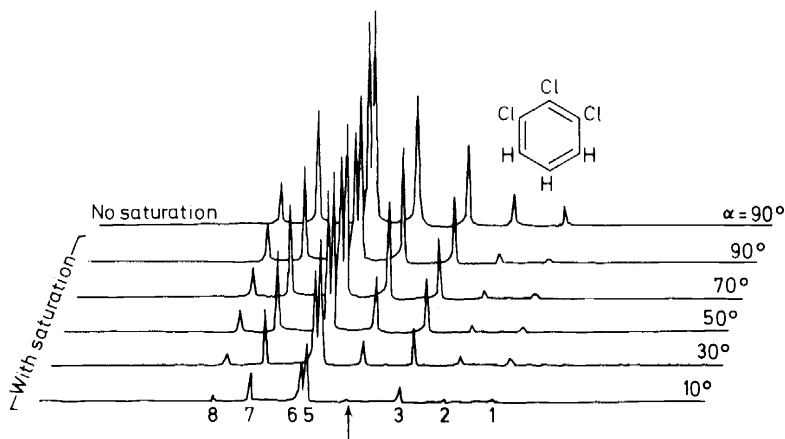Deviations of the populations from a Boltzmann distribution can also

Figure 3. Fourier Overhauser experiment on a 5 per cent molar solution of 1,2,3-trichlorobenzene in carbon disulphide, not degased. Previous to the rf pulse, line 4 was saturated by applying an rf field of an amplitude 1.5 Hz for 30 s. The same experiment was repeated for different flip angles. The proton resonance spectrum was recorded on a Varian DA-60 spectrometer at 60 MHz. 64 measurements were co-added

occur in selective saturation experiments due to generalized Overhauser effects. Experiments of this kind are aimed at determining the relaxation mechanisms in coupled spin systems[22, 23]. This case is formally identical to CIDNP experiments. The same equations apply here as well.

Figure 3 shows an example of a Fourier Overhauser experiment on 1,2,3-trichlorobenzene. Line 4 was saturated by the prolonged application of a radio frequency field. The rf field was removed immediately before applying an rf pulse to excite the free induction decay. Sixty-four measurements were co-added to eliminate transient effects caused by the sudden removal of the saturating rf field.

A comparison of the partially saturated spectrum, with a flip angle of 10°, with the unsaturated spectrum shows the almost complete absence of the saturated line 4. For larger flip angles this line recovers and reaches for $\alpha = 90°$ a relative intensity slightly larger than the unsaturated one. On the other hand, lines 1 and 2 remain saturated at all flip angles.

Again, large flip angles may lead to misleading conclusions unless a careful study of the redistribution of the magnetization during the rf pulse is made. Details of this study will be reported elsewhere.

In this context it should be remembered that analogous problems occur in the measurement of longitudinal relaxation times in strongly coupled systems when the inversion-recovery[6] or saturation-recovery technique[7] is used, since here the 90° pulse generates signals which are not related, in a simple manner, to the populations of the energy levels. Additionally, the recovery of the populations will be non-exponential, in general.

5. FOURIER DOUBLE RESONANCE EXPERIMENTS

Conventional slow passage double resonance experiments usually exhibit at the same time Overhauser polarization effects. as well as changes in the position and number of the resonance lines ('tickling' and 'decoupling' effects). Fourier spectroscopy offers an easy possibility of separating the two independent effects. There are at least three possible Fourier double resonance experiments[24, 25]:

(*i*) The rf field H_2 is applied exclusively before the rf pulse:
Overhauser effects are produced only (Section 4).
(*ii*) The rf field H_2 is applied exclusively after the rf pulse:
Real double resonance effects occur only.
(*iii*) The rf field H_2 is applied continuously:
Both effects occur at the same time.

Case (*iii*) corresponds most closely to the conventional slow passage double resonance technique. But even here differences may be observed. First of all, the Overhauser effects depend, in the manner described in Section 4, on the flip angle. Secondly, it is well known that in a slow passage double resonance experiment signals with the frequencies ω_1, ω_2 and $2\omega_2 - \omega_1$ occur at the detector. The first one of these frequencies is usually observed although it is possible to detect the other frequencies as well[26]. In a Fourier experiment all generated frequencies are detected simultaneously and it is possible to observe, besides the expected resonance frequencies Ω_k, the frequency ω_2 and
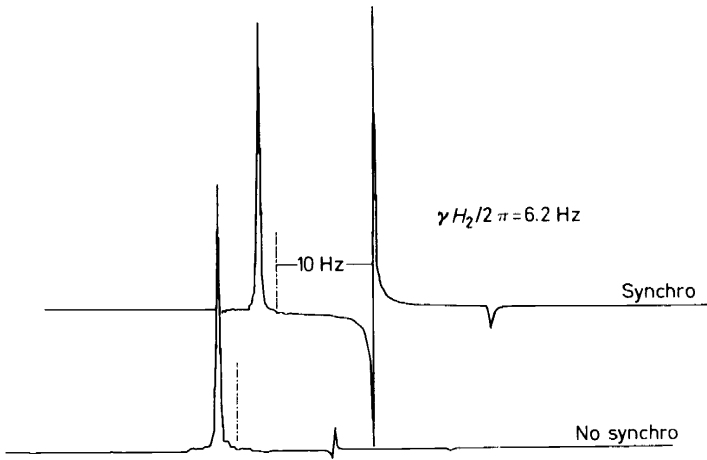
Figure 4. Fourier double resonance experiments of a one-spin system. A constant rf field for 6.2 Hz amplitude was applied 10 Hz to the right of the resonance frequency (indicated by the broken line). For the upper trace the rf pulse was synchronized to the double resonance frequency, whereas in the lower trace no synchronization was employed. 64 scans were co-added

frequencies of the type $2\omega_2 - \Omega_k$. The Fourier double resonance spectrum may show, even in the absence of Overhauser effects, different relative intensities, particularly for strongly coupled spin systems.

In a double resonance experiment the free induction decay consists of frequency components which are phase-related to the double resonance frequency ω_2, and of frequency components which are independent of the phase of ω_2. This offers the possibility of two further modifications of the measurement:
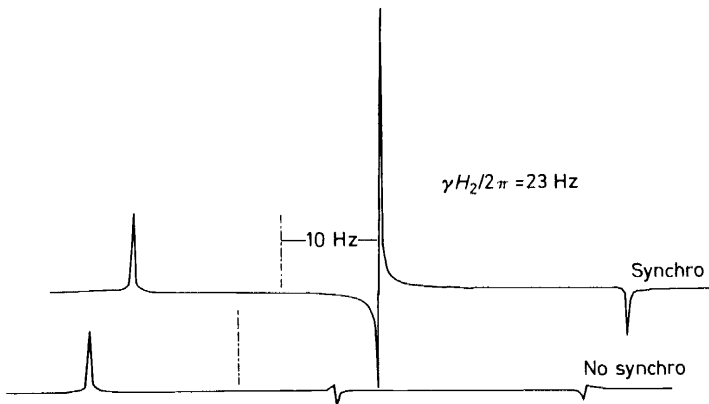

Figure 5. Fourier double resonance experiments of a one-spin system, similar to *Figure 4* except for the rf field of 23 Hz amplitude
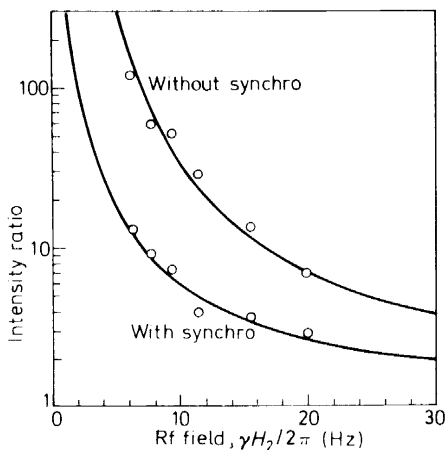
55

Figure 6. Amplitude ratio of the two resonance lines at ω^+ and ω^- as a function of the applied rf field H_2. Two theoretical curves are shown based on equations 18 and 19 for the synchronized and the non-synchronized case. The experimental values were taken from spectra similar to those shown in *Figures 4* and *5*

(*a*) The rf pulse is phase-synchronized to ω_2: In this case all frequency components are observed even when signal averaging is used.

(*b*) The rf pulse is not phase-synchronized to ω_2: Here all frequency components which are phase-related to ω_2 will be averaged to zero in a signal averaging process.

It can be shown that in case (*b*) results will be obtained which are equivalent to a slow passage double resonance experiment. But in case (*a*) significantly different intensities will be obtained.

This may best be illustrated by the almost trivial case of a one-spin system where Overhauser effects are impossible. Here three resonance lines are obtained in the double resonance experiment with the frequencies ω_2 and $\omega_2 \pm [\Delta\omega^2 + (\gamma H_2)^2]^{\frac{1}{2}} = \omega^\pm$. It has been predicted by Krishna and Gordon[27] that the intensity ratio R of the resonances at $\omega^\pm$ is given for case (*a*) by:

$$R^{(a)} = (1 + \cos\theta)/(1 - \cos\theta) \qquad (18)$$

with $\tan\theta = \gamma H_2/(\Omega - \omega_2)$. It can be shown (details will be given elsewhere) that in case (*b*) the same intensities are obtained as in slow passage[28]:

$$R^{(b)} = R^{(SP)} = (1 + \cos\theta)^2/(1 - \cos\theta)^2 \qquad (19)$$

Here the parasitic line is much weaker than in case (*a*).

Experimental examples are shown in *Figures 4* and *5* for two different rf fields. The plot given in *Figure 6* of relative line intensity versus rf field strength verifies the predicted dependence. This effect is independent of the flip angle.

56

6. MODULATION EFFECTS IN FOURIER SPECTROSCOPY

Another reason for a time-dependent density operator equation may be an external modulation of some parameter such as the magnetic field or the rf frequency. Examples are modulation by sample spinning in an inhomogeneous field, modulations caused by the line frequency and purposely generated modulation of the magnetic field H_0. In all these cases the equivalence of Fourier spectroscopy and slow passage will bé violated, in general.

The response of the modulated system again consists of frequency components, phase-related to the modulation mechanism, and of frequency components which are independent of the modulation phase. Signal averaging without synchronization of the rf pulses to the modulation frequency will eliminate all phase-related components. It can be shown again that the remaining Fourier spectrum is proportional to the low-power slow passage spectrum. An analysis of the effects of sample spinning is given in Ref. 29.
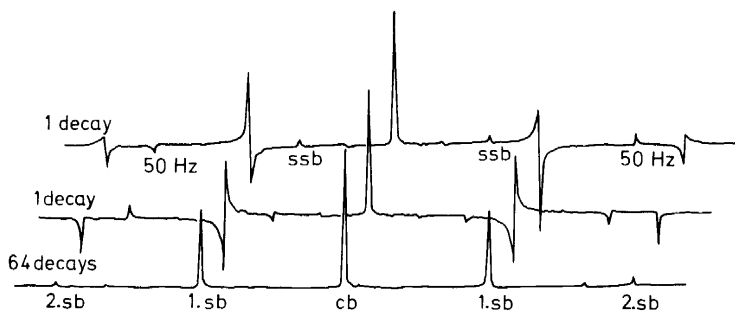

Figure 7. Fourier spectrum of a one-spin system with magnetic field modulation with a modulation frequency of 30 Hz. First and second modulation sidebands (1.sb and 2.sb) are apparent as well as spinning sidebands (ssb) and 50 Hz sidebands. Two examples are given with a single decay and one example with 64 scans signal-averaged

Figure 7 demonstrates all the three mentioned effects. It is seen that for a single decay the phase relations of the various sidebands are unrelated, obviously because of the fact that the rf pulse was applied at an arbitrary instant. On the other hand, in the signal averaged trace, all lines have the same phase as those known from slow passage. The sidebands show a considerable loss of intensity. Particularly the second sidebands are to a large extent phase-related to the modulation frequency and strongly decrease in amplitude upon signal averaging. The same is true for the 50 Hz sidebands. The spinning sidebands are additionally attenuated by the unstable spinning speed.

7. SYSTEMS WITH NON-LINEAR DENSITY OPERATOR EQUATION

There is a number of spin systems which must be described by a non-linear density operator equation. Here again, the equivalence of Fourier spectroscopy and slow passage will be questionable. Examples of practical interest are provided by systems with intermolecular chemical exchange[30, 31] and by systems with intermolecular dipolar relaxation[32, 33].

As a simple example, the system with chemical exchange between two molecules A, B will be described, $A + B \rightleftharpoons A' + B'$. It is characterized by a system of equations of the general type:

$$\dot{\sigma}_A = \Lambda_A \sigma_A + \mathrm{Tr}_B\{\Lambda_{AB}\sigma_A\sigma_B\} + Q_A$$
$$\dot{\sigma}_B = \Lambda_B \sigma_B + \mathrm{Tr}_A\{\Lambda_{BA}\sigma_A\sigma_B\} + Q_B \tag{20}$$

which is clearly a non-linear system of equations, although the operators Λ_A, Λ_B and Λ_{AB} are linear. For a sufficiently high temperature the density operators σ_A and σ_B never deviate appreciably from unity and it is possible to expand them in terms of $\beta = 1/kT$:

$$\sigma_A \approx 1/\mathrm{Tr}\{1\} + \beta\sigma_A^{(1)} + O(\beta^2)$$
$$\sigma_B \approx 1/\mathrm{Tr}\{1\} + \beta\sigma_B^{(1)} + O(\beta^2) \tag{21}$$

and to neglect higher terms in β. This allows the linearization of the density operator equations:

$$\dot{\sigma}_A^{(1)} = \Lambda_A\sigma_A^{(1)} + \mathrm{Tr}_B\{\Lambda_{AB}[\sigma_A^{(1)}/\mathrm{Tr}\{1_B\} + \sigma_B^{(1)}/\mathrm{Tr}\{1_A\}]\} + Q_A^{(1)}$$
$$\dot{\sigma}_B^{(1)} = \Lambda_B\sigma_B^{(1)} + \mathrm{Tr}_A\{\Lambda_{BA}[\sigma_A^{(1)}/\mathrm{Tr}\{1_B\} + \sigma_B^{(1)}/\mathrm{Tr}\{1_A\}]\} + Q_B^{(1)} \tag{22}$$

It is clear that for these equations the equivalence can again be proved provided that the additional conditions (*i*) and (*iii*) (p. 49) are fulfilled as well. Kaplan[31] claims that there may be cases where even at room temperature the high-temperature approximation is violated and that non-linear effects could be observed. There has been some controversy about this point in the literature[34, 35].

The same arguments also apply to intermolecular relaxation mechanisms which also couple the density operator equations of several molecules in a non-linear fashion. Again if the high-temperature approximation is fulfilled the non-linear terms can be safely neglected.

8. CONCLUSIONS

The results may be summarized by the following preliminary table of special cases:

1. High resolution n.m.r. in liquids:
   (*a*) *Single resonance* — Equivalence is always fulfilled as the high-temperature approximation cannot be violated.
   (*b*) *Overhauser experiments* — Equivalence is not fulfilled, except for a very small flip angle.

(c) *Double resonance*—Equivalence is not fulfilled, except for signal averaging and a very small flip angle.

(d) *CIDNP spectra*—Equivalence is not fulfilled, except for a very small flip angle.

(e) *Chemical exchange*—Equivalence is fulfilled.

2. n.m.r. in solids:

(a) *High-field and high-temperature approximations satisfied*—Equivalence holds for any anisotropic dipolar and quadrupolar interactions.

(b) *Low magnetic fields*—Equivalence generally not fulfilled.

(c) *Low temperature*—Equivalence not fulfilled.

(d) *Pure n.q.r.*—Equivalence is fulfilled for spin 1 and for spin 3/2 for sufficiently strong quadrupolar interaction and high temperature.

This demonstrates that in many cases of practical interest, particularly for routine applications, the equivalence is fulfilled. But there is still a large number of important experiments where signal intensities and possibly line shapes depend on the experimental technique. On the other hand, the possible resonance frequencies will always be the same in the two experiments, except for systems with a non-linear density operator equation.

ACKNOWLEDGEMENTS

REFERENCES

[1] R. R. Ernst and W. A. Anderson, *Rev. Sci. Instr.*, **37**, 93 (1966).
[2] R. R. Ernst, *Adv. Magn. Resonance*, **2**, 1 (1966).
[3] T. G. Farrar and E. D. Becker, *Pulse and Fourier Transform NMR*, Academic Press, New York (1971).
[4] D. G. Gillies and D. Shaw, *Ann. Rept. on NMR Spectroscopy*, 5 (1972).
[5] G. C. Levy and G. L. Nelson, *Carbon-13 NMR for Organic Chemists*, Wiley, New York (1972).
[6] R. Freeman and H. D. W. Hill, *J. Chem. Phys.*, **53**, 4103 (1970).
[7] R. Freeman and H. D. W. Hill, *J. Chem. Phys.*, **54**, 3367 (1971).
[8] R. Freeman, H. D. W. Hill and R. Kaptein, *J. Magn. Resonance*, 7, 82 (1972).
[9] G. Horlick and H. V. Malmstadt, *Anal. Chem.*, **42**, 1361 (1970).
[10] E. Bartholdi and R. R. Ernst, *J. Magn. Resonance*, **11**, 9 (1973).
[11] R. Freeman and H. D. W. Hill, *J. Magn. Resonance*, **4**, 366 (1971).
[12] R. R. Ernst and R. E. Morgan, *Mol. Phys.*, **26**, 49 (1973).
[13] D. E. Jones and H. Sternlicht, *J. Magn. Resonance*, 6, 167 (1972).
[14] I. J. Lowe and R. E. Norberg, *Phys. Rev.*, **107**, 46 (1957).
[15] A. Abragam, *The Principles of Nuclear Magnetism* Oxford U.P., London (1961).
[16] E. A. Guillemin, *Theory of Linear Physical Systems*, Wiley, New York (1963).
[17] B. M. Brown, *The Mathematical Theory of Linear Systems*, Science Paperbacks, London (1965).
[18] H. R. Ward, *Accounts of Chem. Res.*, 5, 18 (1972). R. G. Lawler, ibid., 5, 25 (1972).
[19] R. Kaptein *J. Am. Chem. Soc.*, **94**, 6251, 6262, 6280 (1972).
[20] F. J. Adrian, *J. Chem. Phys.*, **54**, 3912 (1971).
[21] S. Schäublin, A. Höhener and R. R. Ernst, *J. Magn. Resonance*, **12** (in press).
[22] R. Freeman, S. Wittekoek and R. R. Ernst, *J. Chem. Phys.*, **52**, 1529 (1970).
[23] N. R. Krishna, P. P. Yang and S. L. Gordon, *J. Chem. Phys.*, **58**, 2906 (1973).
[24] R. Freeman, *J. Chem. Phys.*, **53**, 457 (1970).

[25] F. Günther, *Ann. Phys.*, **7.27**, 396 (1971).
[26] L. R. Anders and J. D. Baldeschwieler, *J. Chem. Phys.*, **43**, 2147 (1965).
[27] N. R. Krishna and S. L. Gordon. *Phys. Rev. A*, **6**. 2059 (1972).
[28] S. L. Gordon, *J. Chem. Phys.*, **45**, 1145 (1966).
[29] R. R. Ernst, *J. Magn. Resonance*, **4**, 280 (1971).
[30] S. Alexander. *J. Chem. Phys.*, **37**, 974 (1962).
[31] J. I. Kaplan, *J. Chem. Phys.*, **57**, 5615 (1972).
[32] T. N. Khazanovich and V. Yu. Zitserman, *Mol. Phys.*, **21**, 65 (1971).
[33] M. Alla and E. Lippmaa, *J. Magn. Resonance*, **4**, 241 (1971).
[34] R. R. Ernst, *J. Chem. Phys.*, **59**, 989 (1973).
[35] J. I. Kaplan, *J. Chem. Phys.*, **59**, 990 (1973).